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On wave function renormalization and related aspects in heavy fermion effective field theories

Sven Steininger*

Physics Department, Harvard University, Cambridge MA 02138, USA
Forschungszentrum Jülich, Institut für Kernphysik (Theorie)
D-52425 Jülich, Germany
E-mail: S.Steininger@fz-juelich.de

Ulf-G. Meißner, Nadia Fettes

Forschungszentrum Jülich, Institut für Kernphysik (Theorie)
D-52425 Jülich, Germany
E-mail: Ulf-G.Meissner@fz-juelich.de, N.Fettes@fz-juelich.de

Abstract: We reconsider the question of wave function renormalization in heavy fermion effective field theories. In particular, we work out a simple and efficient scheme to define the wave function renormalization with respect to the lowest order heavy fermion propagator. The method presented is free of a set of ambiguities which arise in heavy fermion effective field theories. In this context, we discuss the approaches used in the literature so far. We also calculate the fourth order pion mass contribution to the nucleon mass shift and discuss the tree level contributions to the electric Sachs form factor of the nucleon.

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Contents

1. Introduction

Quantum Chromodynamics (QCD) admits two interesting limits, which can be treated by similar methods. In the sector of the heavy quarks (c and b) one observes to leading order the so–called heavy quark and the related spin symmetry [1]. The heavy quarks can be treated in a non–relativistic framework which is called Heavy Quark Effective Field Theory (HQEFT), reminiscent of the well-known Foldy–Wouthuysen transformation for heavy Dirac particles. The QCD Lagrangian for the heavy flavors, collectively denoted by the field Q, takes the simple form

$$\mathcal{L}_{\mathrm{QCD}} = \bar{Q}(i \not{D} - M_Q)Q \,, \tag{1.1}$$

where all the propagation of the heavy quarks and their interactions can be handled as power expansions in $1/M_Q$. Similarly, for the light quark sector, one observes that the current quark masses are small compared to the typical scale of the strong interactions. This defines the chiral limit of QCD, which can be analyzed making use of chiral perturbation theory (CHPT) [2]. This is the effective field theory of the Standard Model at low energies, with the pseudo–Goldstone bosons π, K, η the pertinent degrees of freedom. In the presence of matter fields, like e.g. baryons, a complication arises due to the large baryon mass scale, which is comparable to the scale of chiral symmetry breaking, $m_B \sim \Lambda_\chi \sim 1\,\mathrm{GeV}$. Resorting to methods taken from HQEFT, Jenkins and

JHEP09(1998)008

Manohar [3] showed how to move the troublesome baryon mass term in a string of $1/m_B$ suppressed interaction vertices. This allows for a consistent power counting since the baryon propagator to leading order is given by

$$S(\omega) = \frac{i}{\omega + i\varepsilon} \quad (\varepsilon \to 0^+)\,, \tag{1.2}$$

with $\omega = v \cdot k$ in terms of the four–velocity vector v_μ and the small residual momentum k, $v \cdot k \ll m_B$. As wanted, the baryon mass has disappeared from the propagator. The simplest way to systematically calculate the $1/m_B$ corrections was spelled out in ref.[4], in which the path integral formalism for HQEFT developed by Mannel et al. [5] was extended to baryon CHPT.

Clearly, in heavy baryon CHPT as well as in HQEFT, the massive degrees of freedom behave essentially non–relativistically and it is thus not obvious how to extend the notion of wave function renormalization to such a situation. In relativistic baryon CHPT, this is not an issue since one can apply standard quantum field theoretical methods, as detailed in [2, 6]. In the work of ref. [4], the wave function renormalization was defined via the derivative of the nucleon self–energy at $\omega = 0$, leading to a momentum independent result for Z_N, the heavy nucleon Z–factor. Since the propagator eq. (1.2) develops a pole at this value of ω, it is the natural point to expand around, but one could equally chose other values of ω to define the Z–factor.[1] A somewhat different interpretation was given in ref.[7]. A more detailed analysis of this particular aspect was performed by Ecker and Mojžiš [8], who argued that the Z–factor can not be a constant but rather depends (in momentum space) on the chosen frame via the baryon momentum. They for the first time stressed the role of the heavy fermionic sources and within their scheme, the contribution from these sources is entirely given by Z_N and one thus does not have to perform any explicit calculation for terms involving these heavy sources (once the Z–factor is determined). Note that the Z–factor given in that paper for the "BKKM" approach is not correct, it should be momentum–independent. Such an observation was independently made in ref.[9].[2] This momentum dependence is, however, also present in the treatment à la ref.[4]. In that approach, the tree graphs are calculated from the *relativistic* tree level Lagrangian and then expanded in inverse powers of the nucleon mass up to the needed accuracy. More precisely, one has to include all relativistic tree level Lagrangian terms $\mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \cdots$ which, when expanded, can contribute to the order of M_π/m_N one is after. This procedure contains automatically the momentum dependent pieces through the spinor normalization, as explicitly shown for the case of elastic pion–nucleon scattering in [10]. A general proof that this method always leads to the correct results has not been given. The aim of this paper is to set up a very simple scheme for wave function renormalization in heavy fermion effective field theories which parallels as closely as possible the conventional

[1] We are much indebted to Thomas Hemmert for clarification on this topic.
[2] We are grateful to Gerhard Ecker for confirmation on this statement.

quantum field theory approach. As we will show, it is very useful to elucidate the interrelationship between the various approaches found in the literature. It also provides us with the proof that expanding the relativistic tree graphs indeed leads to the correct result, as first conjectured in [11]. The method developed should also be of interest for HQEFT, since to our knowledge this issue has not been addressed in detail there. An exception to this is ref.[12] in which the question of how to properly normalize spinors in HQEFT is discussed.

The manuscript is organized as follows. In sect. 2 we review the salient features of heavy baryon chiral perturbation theory (HBCHPT) necessary to keep our presentation self–contained. In sect. 3 we establish a novel scheme to define wave function renormalization in heavy fermion EFTs, based on a set of four simple conditions and interpreting the so–called light components of the heavy fermions as Dirac spinors. This scheme is free of a set of ambiguities, which naturally arise in heavy fermion effective field theories. Section 4 is devoted to an alternative interpretation, in that the light components are treated as two–component Pauli spinors. This allows us to establish the relation between the general definiton given before and the method of expanding the relativistic tree graphs, as commonly done [13]. Since S–matrix elements and transition currents do, of course, not depend on the way one defines wave function renormalization, we consider in detail the electric Sachs form factor in sect. 5. We show that all different calculational schemes lead to a constant contribution from the Born terms in conjunction with the appropriate wave function renormalization. Some technicalities and additional remarks are relegated to the appendices.

2. Heavy nucleon effective field theory

We briefly review the path–integral formulation of the chiral effective pion–nucleon system. This follows largely the original work of [4], which was reviewed in [13]. The interactions of the pions with the nucleons are severely constrained by chiral symmetry. The generating functional for Green functions of quark currents between single nucleon states, $Z[j,\eta,\bar\eta]$, is defined via

$$\exp\{i\,Z[j,\eta,\bar\eta]\} = \mathcal{N}\int[du][dN][d\bar N]\;\exp\,i\Big[S_{\pi\pi}+S_{\pi N}+\int d^4x(\,\bar\eta N+\bar N\eta\,)\Big]\;, \quad (2.1)$$

with $S_{\pi\pi}$ and $S_{\pi N}$ denoting the pion and the pion–nucleon effective action, respectively, to be discussed below. η and $\bar\eta$ are fermionic sources coupled to the baryons and j collectively denotes the external fields of vector (v_μ), axial–vector (a_μ), scalar (s) and pseudoscalar (p) type.[3] These are coupled in the standard chiral invariant manner. In particular, the scalar source contains the quark mass matrix $\mathcal{M}$, $s(x)=\mathcal{M}+\cdots$. The underlying effective Lagrangian can be decomposed into a purely mesonic ($\pi\pi$) and a

[3]The external vector field v_μ should not be confused with the four-velocity to be defined later on, which is denoted by the same symbol.

pion–nucleon (πN) part as follows (we only consider processes with exactly one nucleon in the initial and one in the final state)

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} \, , \tag{2.2}$$

subject to the following low–energy expansions

$$\mathcal{L}_{\pi\pi} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \cdots \, , \quad \mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} \cdots \, , \tag{2.3}$$

where the superscript denotes the chiral dimension. The pseudoscalar Goldstone fields, i.e. the pions, are collected in the 2×2 unimodular, unitary matrix $U(x)$, $U(\phi) = u^2(\phi) = \exp\{i\phi/F\}$ with F the pion decay constant (in the chiral limit). The external fields appear in the following chiral invariant combinations: $r_\mu = v_\mu + a_\mu$, $l_\mu = v_\mu - a_\mu$, and $\chi = 2B_0\,(s + ip)$. Here, B_0 is related to the quark condensate in the chiral limit, $B_0 = |\langle 0|\bar{q}q|0\rangle|/F^2$. We adhere to the standard chiral counting, i.e. s and p are counted as $\mathcal{O}(q^2)$, with q denoting a small momentum or meson mass. The effective meson–baryon Lagrangian starts with terms of dimension one,

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left(i\not{D} - m_0 + \frac{\overset{\circ}{g}_A}{2} \not{u}\gamma_5 \right) \Psi \, , \tag{2.4}$$

with m_0 and $\overset{\circ}{g}_A$ denoting the nucleon mass, respectively the axial-vector coupling in the chiral limit and $u_\mu = i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger]$. The nucleons, i.e. the proton and the neutron, are collected in the iso–doublet Ψ, $\Psi^T = (p, n)$. Under $SU(2)_L \times SU(2)_R$, Ψ transforms as any matter field. D_μ denotes the covariant derivative, $D_\mu \Psi = \partial_\mu \Psi + \Gamma_\mu \Psi$ and Γ_μ is the chiral connection, $\Gamma_\mu = \frac{1}{2}\,[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger]$. Note that the first term in eq. (2.4) is of dimension one since $(i\not{D} - m_0)\,\Psi = \mathcal{O}(q)$ [14]. The lowest order pion–nucleon Lagrangian contains two parameters, namely m_0 and $\overset{\circ}{g}_A$. Treating the nucleons as relativistic spin–1/2 fields, the chiral power counting is considerably complicated due to the large mass scale m_0, $\partial_0\,\Psi \sim m_0\,\Psi \sim \Lambda_\chi\,\Psi$, with $\Lambda_\chi \sim 1\,$GeV the scale of chiral symmetry breaking. A detailed analysis of this topic can be found in [6]. This problem can be overcome in the heavy mass formalism proposed in [3]. We follow here the path integral approach developed in [4]. Defining velocity–dependent spin–1/2 fields by a particular choice of Lorentz frame and decomposing the fields into their velocity eigenstates (also called 'light' and 'heavy' fields),

$$H_v(x) = \exp\{im_0 v\cdot x\}\, P_v^+\,\Psi(x)\, , \quad h_v(x) = \exp\{im_0 v\cdot x\}\, P_v^-\,\Psi(x)\, , \tag{2.5}$$

the mass dependence is shuffled from the fermion propagator into a string of $1/m_0$ suppressed interaction vertices. The projection operators appearing in eq. (2.5) are given by

$$P_v^\pm = \frac{1 \pm \not{v}}{2}\, , \quad P_v^+ H = H\, , \; P_v^- h = h\, , \quad P_v^+ + P_v^- = 1\, , \tag{2.6}$$

with v_μ the four–velocity subject to the constraint $v^2 = 1$. To be specific, the nucleon four–momentum has the form

$$p_\mu = m_0\, v_\mu + k_\mu\, , \tag{2.7}$$

where k_μ is a small residual momentum, $v \cdot k \ll m_0$. In the basis of the velocity projected light and heavy fields, the effective pion–nucleon action takes the form

$$S_{\pi N} = \int d^4x \left\{ \bar{H}_v \, A \, H_v - \bar{h}_v \, C \, h_v + \bar{h}_v \, B \, H_v + \bar{H}_v \, \gamma_0 \, B^\dagger \, \gamma_0 \, h_v \right\} . \tag{2.8}$$

The matrices A, B and C admit low energy expansions, e.g.

$$A = A^{(1)} + A^{(2)} + A^{(3)} + A^{(4)} + \cdots , \tag{2.9}$$

and similarly for B and C. Explicit expressions for the various contributions can be found in [13]. Furthermore, we split the baryon source fields $\eta(x)$ into velocity eigenstates,

$$R_v(x) = \exp\{im_0 v \cdot x\} \, P_v^+ \, \eta(x) \, , \quad \rho_v(x) = \exp\{im_0 v \cdot x\} \, P_v^- \, \eta(x) \, , \tag{2.10}$$

and shift variables, $h_v' = h_v - C^{-1} \, (B \, H_v + \rho_v)$, so that the generating functional takes the form

$$\exp[iZ] = \mathcal{N} \, \Delta_h \int [dU][dH_v][d\bar{H}_v] \, \exp\{iS_{\pi\pi} + iS'_{\pi N} \} \tag{2.11}$$

in terms of the new pion–nucleon action $S'_{\pi N}$,

$$\begin{aligned} S'_{\pi N} = \int d^4x \Big\{ & \bar{H}_v (A + \gamma_0 B^\dagger \gamma_0 \, C^{-1} B \,) H_v + \bar{H}_v (R_v + \gamma_0 B^\dagger \gamma_0 C^{-1} \rho_v) + \\ & + (\bar{R}_v + \bar{\rho}_v C^{-1} \, B) H_v + \bar{\rho}_v C^{-1} \, \rho_v \Big\} \, . \end{aligned} \tag{2.12}$$

The determinant Δ_h related to the 'heavy' components is identical to one, i.e. the positive and negative velocity sectors are completely separated. The generating functional is thus entirely expressed in terms of the Goldstone bosons and the 'light' components of the spin–1/2 fields. The action is, however, highly non–local due to the appearance of the inverse of the matrix C. To render it local, one now expands C^{-1} in powers of $1/m_0$, i.e. in terms of increasing chiral dimension. To any finite power in $1/m_0$, one can now perform the integration of the 'light' baryon field components H_v by again completing the square,

$$H_v' = H_v + T^{-1} \, (R_v + \gamma_0 \, B^\dagger \, \gamma_0 \, C^{-1} \, \rho_v \,) \, , \quad T = A + \gamma_0 \, B^\dagger \, \gamma_0 \, C^{-1} \, B \, . \tag{2.13}$$

Notice that the second term in the expression for T only starts to contribute at chiral dimension two. Finally, we arrive at

$$\exp[iZ] = \mathcal{N}' \int [dU] \, \exp\{iS_{\pi\pi} + iZ_{\pi N} \} \, , \tag{2.14}$$

with $\mathcal{N}'$ an irrelevant normalization constant. The generating functional has thus been reduced to the purely mesonic functional. $Z_{\pi N}$ is given by

$$\begin{aligned} Z_{\pi N} = - \int d^4x \Big\{ & \bar{\rho}_v \, (C^{-1} \, B \, T^{-1} \, \gamma_0 \, B^\dagger \, \gamma_0 C^{-1} - C^{-1} \,) \, \rho_v + \\ & + \bar{\rho}_v \, (\, C^{-1} \, B \, T^{-1} \,) \, R_v + \bar{R}_v \, (T^{-1} \, \gamma_0 \, B^\dagger \, \gamma_0 \, C^{-1} \,) \, \rho_v + \\ & + \bar{R}_v \, T^{-1} \, R_v \Big\} . \end{aligned} \tag{2.15}$$

JHEP09(1998)008

At this point, some remarks are in order. First, physical matrix elements are always obtained by differentiating the generating functional with respect to the sources η and $\bar{\eta}$. The separation into the velocity eigenstates is given by the projection operators as defined above. As shown in ref.[8], the chiral dimension of the 'heavy' source $\rho_v \sim P_v^- \eta$ is larger by one order than the chiral dimension of the 'light' source, $R_v \sim P_v^+ \eta$. The effective Lagrangian can be readily deduced from this action. We eschew from giving its explicit form here, see e.g. [13]. At one loop level, divergences appear. These can be extracted either by direct Feynman graph calculations or, more elegantly, directly from the irreducible generating functional [15][16]. Here we give the form relevant to fourth order in SU(2), the details can be found in [17]

$$\begin{aligned} Z_{\rm irr}[j, R_v] = \int d^4x\, d^4x'\, d^4y\, d^4y'\, \bar{R}_v(x)\, S^{\rm cl}_{(1)}(x,y) \Big[\Sigma_2^{(1)+(2)}(y,y')\, \delta(y-y') + \\ + \Sigma_1^{(1)+(2)}(y,y') + \Sigma_3^{(2)}(y,y') \Big]\, S^{\rm cl}_{(1)}(y',x')\, R_v(x') \end{aligned} \tag{2.16}$$

in terms of the self–energy functionals $\Sigma_{1,2,3}$, and $S^{\rm cl}_{(1)}$ denotes the classical propagator in the presence of external fields. Σ_1 refers to the self-energy graphs at order q^3 and the same diagram with one dimension two insertion on the nucleon line. Σ_2 collects the tadpoles at orders q^3 and q^4 and Σ_3 refers to the dimension two vertex corrected self–energy diagrams. The whole machinery and the complete fourth order counterterm Lagrangian is spelled out in [17].

So far, there exist three different approaches how to calculate matrix elements. The first one is based on ref.[4] and will be referred to as "BKKM" in what follows. It amounts to a "hybrid" calculation. The tree graphs are worked out from the relativistic pion–nucleon Lagrangian and then expanded in powers of $1/m_N$ to the order one is interested in. For the reasons mentioned above, the loop graphs are calculated in the heavy nucleon framework. In particular, the light fields H are treated as Pauli spinors and the corresponding Z–factor is entirely given by the loop graphs and is momentum–independent. This method is very convenient for calculations and gives the correct results to orders q^3 and q^4 as will be shown later. The disadvantages of the method are twofold. First, such a hybrid approach does not appeal to everybody and second, it is not clear how it can be extended correctly to higher orders. Second, Ecker and Mojžiš [8] have set up a scheme which stays entirely within the heavy fermion approach, however, matrix elements are matched to the corresponding relativistic ones. This method should be applicable at any order. The derivation rests on the interpretation of the H fields as Dirac spinors. The tree graphs are all calculated within HBCHPT. It is important to note that a *different* Lagrangian is used as compared to BKKM. This Lagrangian is subject to field transformations to eliminate the equation of motion terms and can be found in [18]. In this framework and for the Lagrangian of [18], the wave function renormalization is momentum–dependent (with the exception of forward matrix elements),

$$Z_N(Q) = 1 + \frac{4a_3 M_\pi^2}{m_N^2} + \frac{Q^2}{4m_N^2} + \cdots\,, \tag{2.17}$$

where the residual momentum Q is defined in terms of the on–shell nucleon momentum p_N and its physical mass m_N, $p_N = m_N v + Q$. The LEC a_3 is related to c_1 via $a_3 = m_N\, c_1$. This method has the advantage of staying within one given field theoretical framework, however, the Z–factor they give for the BKKM approach should be momentum–independent. For a more detailed discussion of this approach and the matching to relativistic matrix elements, we refer to ref. [8]. Third, a variant of the BKKM approach, which can easily be extended to higher orders, has been proposed by Fettes et al. [10] (called FMS from here on). Again, the light fields are treated as Pauli–spinors. The tree (Born) graphs are, however, calculated in the heavy baryon limit and the Z–factor consists of two pieces,

$$Z_N = \mathcal{N}^2\, Z_N^{\text{loop}}\,, \tag{2.18}$$

with $\mathcal{N}$ the relativistic spinor normalization,

$$\mathcal{N} = \sqrt{\frac{E_p + m_N}{2m_N}}\,, \tag{2.19}$$

where $E_p = \sqrt{\vec{p}^{\,2} + m_N^2}$ denotes the full relativistic nucleon energy. For the case of pion–nucleon scattering to order $\mathcal{O}(q^3)$, it was demonstrated that this method reproduces the result obtained from expanding the relativistic tree graphs, i.e. the BKKM approach. In fact, as we will show later, one can proove this equivalence quite generally for all one–loop processes including fourth order. This method has the advantage that it also stays within HBCHPT and is thus not a hybrid type of calculation. However, the important normalization factor $\mathcal{N}^2$ which enters the Z–factor, eq. (2.18), is not directly given by the heavy baryon theory. It is important to note that in the rest–frame $v_\mu = (1, \vec{0}\,)$, a connection between the relativistic and the Pauli–spinor interpretation is given through the relation

$$P_v^+\, u = \mathcal{N} \begin{pmatrix} \chi \\ 0 \end{pmatrix}. \tag{2.20}$$

with u (χ) denoting the conventional four(two)–component Dirac (Pauli) spinor. All three methods used so far share one further common disadvantage. One can only calculate the Z–factor if one approaches the physical pole in the direction of the four–velocity v, i.e. when the nucleon four–momentum p defined in terms of the nucleon mass in the chiral limit, m_0, is taken to its on–shell value p_N defined in terms of the physical mass m_N, the difference is $(p_N - p)_\mu \sim v_\mu$. Clearly, it would be preferable to have a definition of the wave function renormalization that does not depend on how one approaches the physical on–shell point. In particular, in the Pauli–spinor interpretation one selects the rest–frame as the preferred frame and that makes the Z–factor depending on this particular choice of the four–velocity vector. The aim of the present work is to set up a scheme which does not have the various short–comings discussed so far and allows to give a very simple and concise definition of wave function renormalization following as closely as possible conventional quantum field theories. Furthermore, as we will demonstrate, in this novel scheme it is particularly easy to compare the existing approaches and pin down the pertinent differences. Clearly, since the Z–factor is not an observable, one is free to chose ones own definition, in particular also the momentum, about which one expands. The only condition to be fulfilled is to work consistently within the chosen framework.

3. Wave function renormalization reconsidered

In this section, we wish to establish a scheme, allowing for a definition of the heavy fermion Z–factor, which we call Z_N from here on, subject to the following four conditions:

1) The definition of Z_N should be independent of the choice of the four–velocity vector v.

2) Its definition should only involve the physical fields.

3) At tree level, one should have $Z_N^{\rm tree} = 1$.

4) The definition of Z_N should be independent of the way one approaches the physical on–shell momentum, $p \to p_N$.

As for the last point, we remark that this is also stressed in ref.[8], however, for the actual calculation a particular "clever" choice was made to control the on–shell limit. While that is certainly legitimate, our aim is to avoid such a choice from the beginning. Although we will establish this scheme for the particular case of two flavor heavy baryon chiral perturbation theory, the method is more general and can be applied to other situations. Of course, we stress again that Z_N is not an observable and that many alternative schemes exist to define it. As we will argue, the one presented here is particularly useful to shed light on the various calculational approaches used so far and helps to clarify the pertinent interrelationships. A precise definition of Z_N is relegated to appendix A.

The starting point of our discussion is the interpretation of the light fields H as *Dirac spinors*, following ref.[8]. In this case, we have to consider all fields in the generating functional. Thus, instead of splitting the sources in light and heavy components, it is advantageous to work with the *physical* external sources. We thus slightly rewrite the generating functional, eq. (2.15), as (the projection operators are kept for clarity)

$$\begin{aligned}\mathcal{Z} = -\int d^4x\, \bar{\tilde\eta}\, \big\{ & P_v^+(A+B'C^{-1}B)^{-1}P_v^+ + P_v^- C^{-1}B(A+B'C^{-1}B)^{-1}P_v^+ + \\ & + P_v^+(A+B'C^{-1}B)^{-1}B'C^{-1}P_v^- + \\ & + P_v^- C^{-1}B(A+B'C^{-1}B)^{-1}B'C^{-1}P_v^- - P_v^- C^{-1}P_v^- \big\}\tilde\eta\,, \qquad (3.1)\end{aligned}$$

with

$$B' := \gamma^0 B^\dagger \gamma^0 \,, \tag{3.2}$$

and the 'tilde' reminds us that the factor $\exp\{im_0 v\cdot x\}$ is still included in the external sources, $\tilde\eta = \exp\{im_0 v\cdot x\}\,\eta$. The physics is given by the Green functions, i.e. the derivatives with respect to the external sources. For S–matrix elements, we only need to concern ourselves with the poles of the Green functions when the particles are on mass shell. In the generating functional, eq. (3.1), only the term $(A+B'C^{-1}B)^{-1}$ can give rise to a pole as long as one considers small external momenta. To specify the most general n–point Green function, we define

$$T := A + B'C^{-1}B = T_0 + T_I \,, \tag{3.3}$$

where the index '0' indicates the absence of external fields and the subsript 'I' denotes the interaction matrix, which to lowest order is nothing but the pion coupling $\sim g_A S\cdot u$. The inverse of T is given by

$$\begin{aligned} T^{-1} &= (T_0+T_I)^{-1} = T_0^{-1}(1+T_I T_0^{-1})^{-1} \\ &= T_0^{-1} - T_0^{-1}\hat T T_0^{-1} = T_0^{-1} - \hat G_n \,. \end{aligned} \tag{3.4}$$

Note that $\hat T$ is nothing but the amputated amplitude for a general process. Stated differently, it amounts to all Born graphs with m interaction vertices and $m-1$ intermediate nucleon propagators. The pertinent n–point Green function G_n thus takes the form

$$\begin{aligned} G_n &= P_v^+\,\hat G_n\,P_v^+ + P_v^-\,C_0^{-1}B_0\,\hat G_n\,P_v^+ + P_v^+\,\hat G_n\,B_0'C_0^{-1}\,P_v^- + \\ &\quad + P_v^-\,C_0^{-1}B_0\,\hat G_n\,B_0'C_0^{-1}\,P_v^- \\ &= \left(1+C_0^{-1}B_0\right)P_v^+\,\hat G_n\,P_v^+\left(1+B_0'C_0^{-1}\right) \\ &= \left(1+C_0^{-1}B_0\right)T_0^{-1}\;P_v^+\,\hat T\,P_v^+\;T_0^{-1}\left(1+B_0'C_0^{-1}\right). \end{aligned} \tag{3.5}$$

To arrive at this result, we have used the commutation relations between the operators A, B and C and the projection operators $P_v^\pm$. We note that eq. (3.5) agrees with eq. (11) of [8]. The S–matrix is now given by reinstating the external legs,

$$\mathcal{S} = \bar u(p')(\not p' - m_N)(1+C_0^{-1}B_0)T_0^{-1}P_v^+\hat T P_v^+ T_0^{-1}(1+B_0'C_0^{-1})(\not p - m_N)u(p)\,. \tag{3.6}$$

Let us first calculate T_0^{-1},

$$\begin{aligned} T_0 &= A_0 + B_0'C_0^{-1}B_0 \\ &= vk + \not k^\perp(2m_0 + vk - 4c_1M^2)^{-1}\not k^\perp + 4c_1M^2 \\ &= (2m_0+vk-4c_1M^2)^{-1}\left(2m_0 vk + k^2 + 8m_0c_1M^2 - 16c_1^2M^4\right) \\ &= (2m_0+vk-4c_1M^2)^{-1}\left(p^2 - m_0^2 + 8m_0c_1M^2 - 16c_1^2M^4\right), \end{aligned} \tag{3.7}$$

where k_μ is the small residual momentum and $k_\mu^\perp = k_\mu - (v \cdot k)v_\mu$ is the momentum orthogonal to the direction given by the four–velocity v. M^2 is the leading term in the quark mass expansion of the pion mass, $M_\pi^2 = M^2[1 + \mathcal{O}(q^2)]$. One can factor out the term C_0^{-1} since it does not have any pole as long as one restricts oneself to small momenta. For a given fixed order, the corrections obtained by this factorization are always of higher order and can thus be neglected. Consequently, only the second term in eq. (3.7) contains a pole, which leads to the well–known mass shift $\delta m = -4c_1 M^2$. So we are left with the calculation of

$$
\begin{aligned}
(\not p - m_N)(1 + C_0^{-1} B_0) T_0^{-1} &= (\not p - m_N)(C_0 + B_0)(p^2 - m_N^2)^{-1} \\
&= (\not p - m_N)(\not p + m_N + vk(1 - \not v))(p^2 - m_N^2)^{-1} \\
&= 1 + 2(\not p - m_N) vk (p^2 - m_N^2)^{-1} P_v^- \, .
\end{aligned} \tag{3.8}
$$

Using furthermore $P_v^- P_v^+ = 0$, the S–matrix follows as (see also appendix A)

$$
\begin{aligned}
\mathcal{S} &= \bar u(p') P_v^+ \hat T P_v^+ u(p) = \bar u(p') Z_N P_v^+ \hat T P_v^+ Z_N u(p) \\
&= \bar u(p')_{\rm phys} \sqrt{Z_N}\, P_v^+ \, \hat T \, P_v^+ \sqrt{Z_N}\, u(p)_{\rm phys} \, ,
\end{aligned} \tag{3.9}
$$

which means that in this case the Z–factor is exactly one, $Z_N = 1$. Of course, there are still corrections from the loops, which will be evaluated subsequently. We stress here that this interpretation allows for a clear and concise definition of the Z–factor in that only the loop graphs lead to a non–trivial contribution.

We now consider the effects of pion loops. For that, we expand around the classical solution of the fermion propagator in terms of pionic fluctuations [6]. This means for the matrices A, B and C defined in eq. (2.8),

$$
\begin{aligned}
A &\to A^{\rm cl} + A^{(1)} + A^{(2)} + \cdots \, , \\
B &\to B^{\rm cl} + B^{(1)} + B^{(2)} + \cdots \, , \\
C &\to C^{\rm cl} + C^{(1)} + C^{(2)} + \cdots \, ,
\end{aligned} \tag{3.10}
$$

where the superscript '(i)' counts the number of pionic fluctuations[4] (for details, see e.g. refs.[6, 15, 16]). Note that we do not need to consider baryonic fluctuations since we only work at small momenta. We now calculate the loop contributions to the Z–factor. We have to generalize eq. (3.5) in that all possible loop effects have to be taken into account. In a symbolic language, this reads

$$
\begin{aligned}
G_n = {} & (1 + \underbrace{C_0^{-1} B_0}) T_0^{-1} \; P_v^+ \, \hat T \, P_v^+ \; T_0^{-1} (1 + B_0' C_0^{-1}) + \\
& + (1 + \underbrace{C_0^{-1} B_0) T_0^{-1}} \; P_v^+ \, \hat T \, P_v^+ \; T_0^{-1} (1 + B_0' C_0^{-1}) + \\
& + (1 + \underbrace{C_0^{-1} B_0) T_0^{-1} \; P_v^+ \, \hat T} \; P_v^+ \; T_0^{-1} (1 + B_0' C_0^{-1}) + \\
& + (1 + \underbrace{C_0^{-1} B_0) T_0^{-1} \; P_v^+ \, \hat T \, P_v^+ \; T_0^{-1}} (1 + B_0' C_0^{-1}) + \\
& + \cdots \, ,
\end{aligned} \tag{3.11}
$$

[4]These should not be confused with the chiral dimension used before.

where the 'underbrace' indicates that loops have to be constructed from the pertinent operator structures involving C_0^{-1} and B_0. A closer look at these contributions reveals that only the first two underbraced combinations of operators have the correct pole structure, however, they only start to contribute at order q^5 (when one counts the small momenta at the level of the Lagrangian). This means that loops involving the heavy sources do indeed only start to contribute at that order. Consequently, for calculating the Z–factor to $\mathcal{O}(q^4)$ we have to work out (in analogy to eq. (3.8))

$$\begin{aligned}(\not p - m_N)&(1 + C_0^{-1}B_0)[T_0 - \Sigma(\omega)]^{-1} = \\ &= (\not p - m_N)(C_0 + B_0)[C_0T_0 - C_0\Sigma(\omega)]^{-1} \\ &= (\not p - m_N)(\not p + m_0 - 4c_1M^2 - 8b_0M^4 + 2vkP_v^-)[C_0T_0 - C_0\Sigma(\omega)]^{-1}\,, \end{aligned} \tag{3.12}$$

with $P_v^+P_v^- = 0$. Z_N is the value at the (physical) pole,

$$\begin{aligned} Z_N^{-1} &= (m_N + m_0 - 4c_1M^2 - 8b_0M^4)^{-1}\frac{d}{dp}[C_0T_0 - C_0\Sigma(\omega)]\Big|_{\rm phys} \\ &= (m_N + m_0 - 4c_1M^2 - 8b_0M^4)^{-1} \times \\ &\quad\times\Big\{2m_N - \frac{m_N}{m_0}(\hat\Sigma^{(3)}(vk_{|\rm phys}) + \hat\Sigma^{(4)}(vk_{|\rm phys})) - \\ &\quad -(2m_0 + vk_{|\rm phys} - 4c_1M^2 - 8b_0M^4)\frac{m_N}{m_0}(\hat\Sigma^{(3)\prime}(vk_{|\rm phys}) + \hat\Sigma^{(4)\prime}(vk_{|\rm phys}))\Big\} \end{aligned} \tag{3.13}$$

$$\begin{aligned} &= \Big(2m_N + \frac{3g_A^2M_\pi^3}{32\pi F_\pi^2} + \mathcal{O}(q^4)\Big)^{-1} \times \\ &\quad\times\Big\{2m_N + \frac{3g_A^2M_\pi^3}{32\pi F_\pi^2} + 2m_N(\frac{3g_A^2M_\pi^2}{32\pi^2F_\pi^2} - \frac{9g_A^2M_\pi^3}{64\pi m_NF_\pi^2}) + \mathcal{O}(q^4)\Big\}\,, \end{aligned} \tag{3.14}$$

where the prime denotes differentiation with respect to p and we used

$$\frac{d}{dp}vk = \frac{m_N}{m_0} = 1 + \mathcal{O}(q^2)\,. \tag{3.15}$$

The expressions for the self–energies $\hat\Sigma^{(3,4)}$ are given in appendix B. Putting pieces together, we arrive at

$$Z_N = \Big\{1 - \frac{3g_A^2M_\pi^2}{32\pi^2F_\pi^2} + \frac{9g_A^2M_\pi^3}{64\pi m_NF_\pi^2}\Big\} + \mathcal{O}(q^4)\,. \tag{3.16}$$

Note that the first correction from two loops appears at order q^4 since the Z–factor is the derivative of the self–energy and thus one power of small momentum is absorbed in the definition of Z_N. We remark that the Z–factor is momentum–independent and that, by construction, it fulfills all four requirements spelled out in the beginning of this section. We are now in the position to analyze the various methods used so far in explicit HBCHPT calculations, mostly based on a Pauli–spinor interpretation of the light fields. To end this section, we note that in ref.[8], the Dirac interpretation was also used. In contrast to what is done here, field redefinitions have been used to bring the effective Lagrangian into a minimal form. We remark that the question of the influence of such field redefinitions needs to be addressed in more detail. Such an investigation is underway but goes beyond the scope of the present paper.

4. Pauli spinor interpretation

To clarify the approach used by BKKM and also by FMS, we now interpret the light fields H as two–component Pauli spinors. Clearly, this differs from the interpretation given in the preceeding section and the crucial point will be the discussion of the contribution from the heavy fields/sources, which in BKKM are hidden in the expansion of the relativistic tree graphs and in FMS come in via the spinor normalization factor.

To be more precise, we reiterate the statement already made in sect. 2 that in the rest–frame $v_\mu = (1, \vec{0}\,)$, a connection between the relativistic and the Pauli–spinor interpretation is given through the relation $P_v^+ \, u = \mathcal{N}\,(\chi, 0)^T$, with u (χ) denoting the conventional four(two)–component Dirac (Pauli) spinor. In light of these remarks, consider again the generating functional. In the Pauli spinor interpretation, the pion–nucleon action has no heavy sources, cf. eq. (2.12),

$$S'_{\pi N} = \int d^4x \, \bar{H}_v (A + B' \, C^{-1} B\,) H_v + \bar{H}_v \, R_v + \bar{R}_v \, H_v \; . \tag{4.1}$$

Completing the square is achieved by setting

$$H'_v = H_v - T^{-1} \, R_v \, , \quad T = A + B' \, C^{-1} \, B \, , \tag{4.2}$$

leading to

$$\mathcal{Z} = - \int d^4x \bar{R}_v (A + B' C^{-1} B)^{-1} R_v \, , \tag{4.3}$$

which means that the other components do not play any role. The matrices A,B and C are the standard ones when the relativistic Lagrangian is turned into its heavy fermion form as explained in section 2. Consequently, the inverse propagator is given by

$$S^{-1} = A_0 + B'_0 C_0^{-1} B_0 = T_0 \, , \tag{4.4}$$

with T_0 given in eq. (3.7). Therefore, to this order one again obtains the standard mass shift $\delta m = -4c_1 M^2$. The loop corrections to this result will be discussed below. It is instructive to first consider the Z–factor, which is defined as follows:

$$\begin{aligned} Z_N^{-1} &:= \frac{d}{dp} S^{-1} \Big|_{\not p = m_N} \\ &= (2m_0 + vk - 4c_1 M^2)^{-1} \Big|_{\text{phys}} \cdot \frac{d}{dp} \left(p^2 - m_N^2 \right) \Big|_{\not p = m_N} \\ &= 2m_N (m_N + vp)^{-1} \, , \end{aligned} \tag{4.5}$$

where "phys" means that the expression has to be evaluated at the physical value of vk. Setting now $v = (1, 0, 0, 0)$ (i.e. considering the rest–frame of the heavy nucleon) gives

$$Z_N = \frac{m_N + E_p}{2m_N} = \mathcal{N}^2 \, , \tag{4.6}$$

with $\mathcal{N}$ the normalization factor of the relativistic spinors, already given in eq. (2.19). This underlines the observation made by Fettes et al. [10], namely that retaining the spinor normalization allows to recover the expanded full relativistic tree result directly from the heavy nucleon approach (in that paper, pion–nucleon scattering was investigated). Here, this observation is clearly more general. In fact, the definition of the Z–factor given here fully justifies the method used by BKKM (at least to one loop order q^3 and q^4. Higher order calculations have not yet been attempted, with the exception of [19] [20] and [9]).

The self–energy calculation including the one loop effects proceeds as outlined before. We only want to stress that in the Pauli spinor interpretation one has the exact same pole and thus the same mass shift as in the Dirac case discussed in the previous section. The Z–factor is, however, different,

$$
\begin{aligned}
Z^{-1} &= (E_p + m_0 - 4c_1 M^2 - 8b_0 M^4)^{-1} \Big\{ 2m_N - \frac{m_N}{m_0}(\hat{\Sigma}^{(3)}(vk_{|\mathrm{phys}}) + \hat{\Sigma}^{(4)}(vk_{|\mathrm{phys}})) - \\
&\quad -(2m_0 + vk_{|\mathrm{phys}} - 4c_1 M^2 - 8b_0 M^4)\frac{m_N}{m_0}(\hat{\Sigma}^{(3)\prime}(vk_{|\mathrm{phys}}) + \hat{\Sigma}^{(4)\prime}(vk_{|\mathrm{phys}})) \Big\} \qquad (4.7) \\
&= \left(E_p + m_N + \frac{3g_A^2 M_\pi^3}{32\pi F_\pi^2} + \mathcal{O}(q^4) \right)^{-1} \times \\
&\quad \times \left\{ 2m_N + \frac{3g_A^2 M_\pi^3}{32\pi F_\pi^2} + 2m_N\Big(\frac{3g_A^2 M_\pi^2}{32\pi^2 F_\pi^2} - \frac{9g_A^2 M_\pi^3}{64\pi m_N F_\pi^2}\Big) + \mathcal{O}(q^4) \right\}, \qquad (4.8)
\end{aligned}
$$

and thus

$$
Z_N = \frac{E_p + m_N}{2m_N} \left\{ 1 - \frac{3g_A^2 M_\pi^2}{32\pi^2 F_\pi^2} + \frac{9g_A^2 M_\pi^3}{64\pi m_N F_\pi^2} \right\} + \mathcal{O}(q^4)\,. \qquad (4.9)
$$

Comparison with eq. (3.16) gives

$$
Z_N^{\mathrm{Pauli}} = \mathcal{N}^2\, Z_N^{\mathrm{Dirac}}\,. \qquad (4.10)
$$

This shows that the factorization, which appeared somewhat *ad hoc* in eq. (2.18), is exactly reproduced. It has its origin in the precise matching of the heavy baryon to the fully relativistic theory. This is mirrored in the momentum–dependence of Z_N in eq. (4.9) and it justifies *a posteriori* the methods employed by BKKM (at one loop order) and FMS. A precise statement about what happens at higher orders can only be made when one performs explicit calculations. This is not the aim of this paper. We finally remark that for HBCHPT, the Pauli interpretation can be considered the natural framework.

5. The charge form factor of the nucleon

In this section, we explicitly calculate the isovector electric Sachs form factor of the nucleon to third order in the chiral expansion. This serves to illustrate the point that all methods discussed so far lead to the same result if applied correctly. It also shows in a very transparent way the differences of the various frameworks in intermediate steps. A detailed discussion of the nucleon form factors can be found in [21].

To be specific, consider the nucleon matrix element of the isovector component of the quark vector current $V^i_\mu = \bar{q}\gamma_\mu(\tau^i/2)q$ in the Breit frame. It was already shown in ref.[4] that in HBCHPT this is the natural frame (since the nucleon essentially behaves as brick–wall with respect to the incomimg soft virtual photon). In the rest–frame $v_\mu = (1, \vec{0}\,)$, the matrix–element of the isovector–vector current takes the form (note that the superscript $'v'$ refers to the isovector current, not the four–velocity)

$$\langle N(p')|V^i_\mu(0)|N(p)\rangle = \chi^\dagger_2 \left[G^v_E(k^2)v_\mu + \frac{1}{m_N} G^v_M(k^2)[S_\mu, S_\nu]k^\nu \right] \chi_1 \times \eta^\dagger_2 \frac{\tau^i}{2}\eta_1 \, , \qquad (5.1)$$

where χ is a Pauli spinor with isospin component η and $k^2 = (p' - p)^2 < 0$.[5] is the invariant momentum transfer squared $G^v_E(k^2)$ and $G^v_M(k^2)$ are the isovector electric and magnetic Sachs form factors. We remark that we can replace the Pauli–Lubanski spin–vector S_μ by the Pauli spin matrices, since $S_\mu = (\vec{\sigma}, \vec{\sigma}\,)/2$ is restricted to the two upper components. Obviously, we also need this matrix element sandwiched between Dirac spinors, in which case it takes the form

$$\begin{aligned} &\langle N(p')|V^i_\mu(0)|N(p)\rangle = \\ &= \bar{u}(p')P^+_v \left[\tilde{G}^v_E(k^2)v_\mu + \frac{1}{m_N} \tilde{G}^v_M(k^2)[S_\mu, S_\nu]k^\nu \right] P^+_v u(p) \times \eta^\dagger_2 \frac{\tau^i}{2}\eta_1 \, , \end{aligned} \qquad (5.2)$$

where $u(p)$ is a Dirac spinor. Here, $\tilde{G}^v_E(k^2)$ and $\tilde{G}^v_M(k^2)$ are related to the isovector electric and the isovector magnetic Sachs form factor, via

$$\tilde{G}^v_{E,M} = \frac{1}{\mathcal{N}_1\mathcal{N}_2} G^v_{E,M} = \frac{2m_N}{E + m_N} G^v_{E,M} \, . \qquad (5.3)$$

We are now in the position to calculate these form factors. Note that we calculate the objects $\tilde{G}^v_{E,M}$ within the Dirac spinor framework whereas the $G^v_{E,M}$ follow in the Pauli spinor interpretation. From here on, we concentrate on the charge (electric) isovector form factor. The pertinent kinematics for the tree level photon–nucleon coupling, cf. fig.1, is

$$\begin{aligned} &p = m_0\, v + p_1 \, , \quad p' = m_0\, v + p_2 \, , \quad p^\mu_1 = \left(E', -\frac{\vec{k}}{2} \right) , \quad p^\mu_2 = \left(E', +\frac{\vec{k}}{2} \right) , \\ &k^\mu = \left(0, \vec{k}\, \right) , \quad v \cdot k = 0 \, , \quad v \cdot p_1 = v \cdot p_2 = E' = E - m_0 = \mathcal{O}(q^2) \, , \end{aligned} \qquad (5.4)$$

where E' denotes the residual energy and $p_{1,2}$ the soft residual momenta of the in– and out–going nucleon, respectively. We can rewrite the isovector Sachs form factors as

$$G^v_{E,M}(k^2) = Z_N \cdot (\text{Born terms} + \text{loops}) \, , \qquad (5.5)$$

[5]The photon four–momentum k should not be confused with the same symbol previously denoting the nucleons' small residual momentum.

with the loop contribution being the same in all approaches. Here, we concentrate on the Born (tree) terms and the Z–factor, which are different in all schemes but when combined, should lead to the same result, symbolically

$$G_E^{v,\mathrm{Born}}(k^2) = e\,, \tag{5.6}$$

as it follows from the relativistic calculation [6].

Note that from the Born terms we only consider the ones with fixed coefficients in the $1/m_N$ expansion since all others contribute to the anomalous magnetic moment, the charge and magnetic radii and so on. We now calculate explicitly the Born contribution to $G_E^v(k^2)$ to third order in small momenta within the Pauli spinor interpretation (which is equivalent to FMS approach to this order). This is given by all tree level terms free of LECs that can contribute at third order to the generic diagram shown in fig. 1.

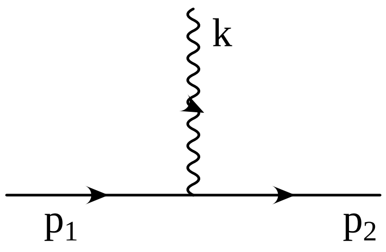

Figure 1: Generic tree diagram for the photon coupling to the nucleon.

The Born terms corresponding to the electric Sachs form factor give

$$\begin{aligned}\mathrm{Born\,terms} &= e\left(1 + \frac{3E'^2}{4m_N^2} - \frac{1}{8m_N^2}\vec{k}^{\,2} - \frac{1}{8m_N^2}(p_1^2 + p_2^2 + 4E'^2) + \frac{1}{16m_N^3}\,E'\,\vec{k}^{\,2}\right)\\ &= e\left(1 - \frac{\vec{k}^{\,2}}{16m_N^2}\right) + \mathcal{O}(q^4)\,, \end{aligned}\tag{5.7}$$

since $p_1^2 = p_2^2 = E'^2 - \vec{k}^{\,2}/4$ and E' is of chiral order two. Expanding the Born Z–factor, see eq. (4.6), to third order in momentum,

$$Z_N^{\mathrm{Pauli}} = \mathcal{N}^2 = 1 + \frac{\vec{k}^{\,2}}{16m_N^2} + \cdots\,, \tag{5.8}$$

we finally get for the isovector electric Sachs form factor,

$$G_E^{v,\mathrm{Born}}(k^2) = \mathcal{N}^2 \cdot (\mathrm{Born\,terms}) = e\,(1 + \mathcal{O}(q^4))\,, \tag{5.9}$$

i.e. to the order one is working, the tree graphs give a constant form factor with the correct charge. We now turn to the BKKM approach. It formally amounts to calculate

$$G_E^{v,(\mathrm{BKKM})}(k^2) = Z_N^{\mathrm{BKKM}} \cdot (\mathrm{rel.\ Born\ terms} + \mathrm{loops})\,. \tag{5.10}$$

As already stated, the calculation of the Born terms is trivial since the relativistic Born graphs lead to a constant $G_E^v(k^2)$ and thus for this application, the calculation is considerably simpler than using HBCHPT to work out the tree graphs in the Pauli spinor interpretation as detailed above.

Finally, we are left with the EM version. Here, the Born terms take yet another form since they start from a different Lagrangian,

$$(\text{Born terms})^{\text{EM}} = e\left(1 + \frac{vp}{m_0} + \frac{k^2}{8m_0}\right), \tag{5.11}$$

with $vp = \delta m - k^2/(8m_N^2) + \cdots$. Multiplying this with the appropriate Born term Z–factor,

$$Z_N^{-1} = \frac{d}{dp}S^{-1} = \frac{m_N}{m_0}\left(1 - \Sigma^{(3)\prime}(0)\right) + \mathcal{O}(q^4), \tag{5.12}$$

one finds that due to the term $\sim m_0/m_N$ the charge form factor is again constant and properly normalized. We add a few remarks: First, the difference between the BKKM and FMS schemes is $\delta G_E^v \sim (Z_N^{\text{BKKM}} - Z_N^{\text{FMS}})\cdot$loops $\sim \mathcal{O}(q^4)$, i.e. one would probably have to modify the BKKM scheme for the wave function renormalization when it comes to two–loop (or higher order) calculations. We do not follow this subtlety here in detail, but it should be kept in mind. Second, we turn our attention to the Dirac spinor interpretation, which allows for comparison of all schemes. The calculation of the Sachs form factors can be summarized as follows:

$$\begin{aligned} G_{E,M}^{v,\,\text{Dirac}}(k^2) &= \frac{E+m_N}{2m_N}\tilde{G}_{E,M}^{v,\,\text{Dirac}}(k^2) \\ &= \frac{E+m_N}{2m_N} Z_N^{\text{Dirac}} \cdot (\text{Born terms} + \text{loops}) \\ &= Z_N^{\text{Pauli}} \cdot (\text{Born terms} + \text{loops}) \\ &= G_{E,M}^{v,\,\text{Pauli}}(k^2) \end{aligned} \tag{5.13}$$

which means that the Pauli und the Dirac interpretations give exactly the same result.

In summary, there are many ways to arrive at the correct result, however, given a certain scheme, one strictly has to stay within its rules. It is a straightforward exercise to show that the loops do indeed not renormalize the charge, as it follows from gauge invariance, but only lead to a momentum dependence of the electric Sachs form factor. For the sake of brevity, we do not spell out the pertinent details here.

6. Summary and conclusions

In this paper, we have considered the questions surrounding wave function renormalization in heavy fermion effective field theories. As an example, we have studied heavy baryon chiral perturbation theory for two flavors and in the isospin limes. The pertinent results of this investigation can be summarized as follows:

i) The most natural and economic way of defining wave function renormalization in heavy fermion effective field theories rests on the interpretation of the light components of the heavy fields (like e.g. the nucleons) as *Dirac* spinors. In that way, one can define the Z–factor subject to four conditions as detailed in section 3. In particular, the so–defined Z–factor is momentum–independent and its tree graph contribution is equal to one. Furthermore, this prescription can be extended to higher orders, i.e. beyond one loop, without problems. Such an interpretation is mandated by the correct matching of the heavy fermion EFT to its relativistic counterpart.

ii) All calculations performed so far in HBCHPT have been done under the assumption that the light components of the heavy fields are to be interpreted as *Pauli* spinors. We have shown the equivalence between such an approach and the Dirac spinor interpretation, provided one works in the rest–frame $v = (1, \vec{0}\,)$ in the Pauli case. This allows to *justify* a posteriori the methods employed by BKKM ($1/m_N$ expansion of the relativistic tree graphs independent of the spinor interpretation) and by FMS (inclusion of the explicit four–dimensional spinor normalization in the tree graphs calculated from HBCHPT). We also pointed towards some potential complications which arise when one employs field redefinitions.

iii) When applied correctly, all these different schemes lead to the same physics. As an example, we have shown how the tree result for the proton charge form factor, $G_E^{\rm tree}(k^2) = e = {\rm const.}$, with e the proton charge and k^2 the photon four–momentum squared, emerges in the various calculational schemes. We have also discussed the non–renormalization of the electric charge due to the loop graphs.

iv) Furthermore, we have studied the nucleon mass shift to fourth order in the pion mass. Apart from an unspecified counter term contribution, which formally amounts to a quark mass correction of a dimension two operator, these corrections are tiny.

We hope that with this paper, the long–standing question of wave function renormalization in heavy fermion effective field theories can finally be put to rest.

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A. Precise definition of the Z–factor

We first repeat the standard steps to get from the generating functional $\mathcal{Z}[j,\eta,\bar\eta]$ for bosonic fields (like e.g. the pions) and fermions (like e.g. the nucleons) coupled to $\bar\eta$ and η in the presence of some external sources j (like e.g. photons or quark mass insertions) to the S–matrix (to be precise, we consider processes with one fermion line running through the pertinent Feynman graphs):

1. Differentiation with respect to the external sources (here: $j,\eta,\bar\eta$).
2. Multiplication with the physical propagator.
3. Multiplication with the spinors u and $\bar u$.
4. Substitution of $\sqrt{Z}u$ by u_{phys} and similarly for $\bar u$.

This leads to the form of eq. (3.9). Thus, Z_N is defined by the product of the physical propagator and the two–point Greens function. For our case, this leads to

$$\begin{aligned} Z_N &:= (\not p - m_N)(1 + C_0^{-1}B_0)P_v^+ T_0^{-1} P_v^+ (1 + B_0' C_0^{-1}) \\ &= (1 + C_0^{-1}B_0)T_0^{-1}P_v^+ + (1 + C_0^{-1}B_0)T_0^{-1}B_0'C_0^{-1}P_v^- \\ &= P_v^+ + \frac{E_p}{m_N}P_v^- \\ &= Z_N^+ P_v^+ + Z_N^- P_v^- \,, \end{aligned} \tag{A.1}$$

which means that the Z–factor appears as the sum of two terms, which are proportional to the velocity projection operators. The part of relevance for the present discussion is, of course, Z_N^+, since all Greens functions corresponding to the positive velocity eigenstates include a projector P_v^+. This is exactly what has been used in eqs. (3.8), (3.12).

B. Self–energy and mass shift of the nucleon to fourth order

In this appendix, we consider the nucleon self–energy and mass shift to fourth order in the Dirac framework (which have not been given in the literature and are needed to arrive at the final expression for Z_N). To calculate the corresponding mass shift, we only need to work out the influence of the pionic fluctuations at the pole,

$$\begin{aligned} T_0^{-1} &= [T_0^{\rm cl} + T_0^{(1)} + T_0^{(2)} + \cdots]^{-1} \\ &= S^{\rm cl} - S^{\rm cl}T_0^{(1)}S^{\rm cl} - S^{\rm cl}T_0^{(2)}S^{\rm cl} + S^{\rm cl}T_0^{(1)}S^{\rm cl}T_0^{(1)}S^{\rm cl} + \cdots \,, \end{aligned} \tag{B.1}$$

where $(S^{\rm cl})^{-1} = T_0^{\rm cl}$. Integrating over the fluctuation fields, the second term in eq. (B.1) vanishes, whereas the third and the fourth give the tadpole and the self–energy contribution, respectively. We can thus bring the expression for the self–energy into a compact form

$$T_0^{-1} = S^{\rm cl} + S^{\rm cl}\Sigma S^{\rm cl} + \cdots = [T_0^{\rm cl} - \Sigma]^{-1} \,. \tag{B.2}$$

The last equation is exact in the sense that if one were to calculate the contributions from all irreducible graphs to Σ, the reducible ones follow from the geometric series (as it is well–known). The inverse propagator follows as

$$\begin{aligned} S^{-1} &= vk + k^{\perp}(2m_0 + vk - 4c_1 M^2 - 8b_0 M^4)^{-1} k^{\perp} + \\ &\quad +4c_1 M^2 + 8b_0 M^4 - \hat{\Sigma}^{(3)}(vk) - \hat{\Sigma}^{(4)}(vk) \\ &= (2m_0 + vk - 4c_1 M^2 - 8b_0 M^4)^{-1} \times \\ &\quad \times \Big\{ p^2 - m_0^2 + 2m_0(4c_1 M^2 + 8b_0 M^4) - (4c_1 M^2 + 8b_0 M^4)^2 - \\ &\quad -(2m_0 + vk - 4c_1 M^2 - 8b_0 M^4)(\hat{\Sigma}^{(3)}(vk) + \hat{\Sigma}^{(4)}(vk)) \Big\} , \end{aligned} \tag{B.3}$$

with $\hat{\Sigma}^{(3,4)}(\omega)$ denoting the third and fourth order loop contribution to the nucleons' self–energy. Here, b_0 is a combination of the LECs of three q^4 counter terms, which in fact lead to a quark mass correction of the LEC c_1. The numerical value of b_0 is at present not known. It could be obtained from a fourth order analysis of the baryon masses and σ–terms (for a model–dependent determination within SU(3) baryon CHPT, see e.g. [19]). In what follows, we absorb the contribution of the b_0 term via

$$c_1' \, M_\pi^2 = c_1 \, M^2 + 2b_0 M^4 \, . \tag{B.4}$$

Explicit calculation using the usual procedure of renormalization in CHPT [2], leads to the third order contribution to Σ,

$$\begin{aligned} \hat{\Sigma}^{(3)}(\omega) &= \Sigma^{(3)}_{\rm loop}(\omega) + \Sigma^{(3)}_{\rm div}(\omega) \\ &= \frac{3g_A^2}{4F_\pi^2}(M_\pi^2 - \omega^2)\left(\frac{\omega}{8\pi^2} - \frac{1}{4\pi^2}\sqrt{M_\pi^2 - \omega^2}\arccos\frac{-\omega}{M_\pi}\right) - \\ &\quad -\frac{3g_A^2}{2F_\pi^2}(3M_\pi^2 - 2\omega^2)\frac{\omega}{16\pi^2}\ln\frac{M_\pi}{\lambda} + \omega^3 d_{24}^r(\lambda) - 8M_\pi^2\omega d_{28}^r(\lambda) \, . \end{aligned} \tag{B.5}$$

Similarly, the fourth order self–energy reads

$$\begin{aligned} \hat{\Sigma}^{(4)}(\omega) &= \Sigma^{(4)}_{\rm loop}(\omega) + \Sigma^{(4)}_{\rm div}(\omega) - \\ &= -\frac{3g_A^2}{8m_N F_\pi^2}\left(k^2 - 2\omega^2 + 8m_N c_1 M_\pi^2 + M_\pi^2\right) \times \\ &\quad \times \left(\frac{1}{8\pi^2}(\omega^2 + M_\pi^2) - \frac{3\omega}{4\pi^2}\sqrt{M_\pi^2 - \omega^2}\arccos\frac{-\omega}{M_\pi}\right) + + \frac{3}{128\pi^2 F_\pi^2}c_2 M_\pi^4 - \\ &\quad -\frac{9g_A^2}{4m_N F_\pi^2}(k^2 - 2\omega^2 + 8m_N c_1 M_\pi^2 + M_\pi^2)(M_\pi^2 - 2\omega^2)\frac{1}{16\pi^2}\ln\frac{M_\pi}{\lambda} + \\ &\quad + \left(12c_1 M_\pi^4 - 6c_3 M_\pi^4 - \frac{3M_\pi^4}{2}\left(c_2 - \frac{g_A^2}{8m_N}\right)\right)\frac{1}{16\pi^2 F_\pi^2}\ln\frac{M_\pi}{\lambda} + \\ &\quad + \frac{9g_A^2 M_\pi^4}{256\pi^2 m_N F_\pi^2}\ln\frac{M_\pi}{\lambda} - 16M_\pi^4 b_{21}^r(\lambda) - 4M_\pi^2\omega^2 b_{160}^r(\lambda) - \\ &\quad -4M_\pi^2 k^2 b_{161}^r(\lambda) - \omega^4 b_{197}^r(\lambda) - \omega^2 k^2\left(b_{198}^r(\lambda) + b_{199}^r(\lambda)\right) , \end{aligned} \tag{B.6}$$

with λ the scale of dimensional regularization. All quantities have been set to their physical values since the differences to the chiral limit values only appear at next order. Of course, eq. (B.5) agrees with the third order self–energy expression given in [4]. The logarithms in eq. (B.6) stem from the self–energy graphs (fourth last) and the tadpoles (third last line), respectively. The $b^r_{21,\ldots,199}$ are renormalized fourth order LECs taken from table 1 of ref.[17] (note that the fourth order LECs are called d_i in [17]. To avoid confusion with the labelling in the FMS Lagrangian, we call them b_i here). By a proper redefinition of the renormalized fourth order LECs, one can absorb all the logarithmic terms,

$$b^r_i(\lambda) = \frac{\beta_i}{16\pi^2}\left[\bar{b}_i + \ln\frac{M}{\lambda}\right]. \tag{B.7}$$

Consequently, all $\ln(M_\pi/\lambda)$ terms vanish in eq. (B.6). The corresponding $\bar{d}_i$ and $\bar{b}_i$ are all zero, as detailed in [10]. With a similar procedure for the d_i, one can also absorb all the logarithms in eq. (B.5), for details see [10]. From the pole position we read off the mass shift

$$\begin{aligned}\delta m = &-\frac{2m_0}{2m_0+\delta m}4c_1'\,M_\pi^2 + \frac{1}{2m_0+\delta m}4c_1'\,M_\pi^2 + \\ &+\frac{2m_0+vk-4c_1'\,M_\pi^2}{2m_0+\delta m}\left(\hat{\Sigma}^{(3)}(vk)+\hat{\Sigma}^{(4)}(vk)\right)\Big|_{\rm phys}\,,\end{aligned} \tag{B.8}$$

To proceed, we have to work out the value $vk|_{\rm phys}$,

$$vk|_{\rm phys} = \frac{p^2-m_0^2-k^2}{2m_0} = 0+\delta m - \frac{k^2}{2m_N}+\cdots\,. \tag{B.9}$$

Let us comment on the k^2–dependent terms. The third order self–energy $\Sigma^{(3)}$ does not depend on vk, since $vk=\mathcal{O}(q^2)$, i.e. such terms can not contribute at third order. At fourth order, however, one gets a term of the type $vk\,\Sigma^{(3)\prime}(0)$. The nucleon mass shift is, of course, not k^2–dependent. At fourth order, all terms $\sim k^2$ cancel, as we will make explicit below. The fourth order mass shift reads

$$\begin{aligned}\delta m^{(4)} = &\frac{3M_\pi^4 c_2}{128\pi^2F_\pi^2} - \frac{3g_A^2M_\pi^4}{64\pi^2m_NF_\pi^2} - \\ &-\frac{3M_\pi^4}{32\pi^2F_\pi^2}(-8c_1+c_2+4c_3)\ln\frac{M_\pi}{\lambda} - \frac{3g_A^2M_\pi^4}{32\pi^2m_NF_\pi^2}\ln\frac{M_\pi}{\lambda} + \\ &+16M_\pi^4\left(2c_1d_{28}^r-b_{21}^r\right)+4M_\pi^2k^2\left(\frac{d_{28}^r}{m_0}-b_{161}^r\right),\end{aligned} \tag{B.10}$$

with the pertinent β–functions, $(4\pi F)^2\,\beta(b^r_{161}) = -9g_A^2/(16m_0)$ and $(4\pi F)^2\,\beta(d^r_{28}) = -9g_A^2/16$. As promised, the k^2 terms add up to zero. In that basis, the mass shift to fourth order takes the form

$$\delta m = -4c_1'M_\pi^2 - \frac{3g_A^2M_\pi^3}{32\pi F_\pi^2} + \frac{3M_\pi^4c_2}{128\pi^2F_\pi^2} - \frac{3g_A^2M_\pi^4}{64\pi^2m_NF_\pi^2} + \mathcal{O}(q^5)\,. \tag{B.11}$$

Here, all masses and couplings are set to their physical values, the error made by this procedure is of higher order. It is instructive to work out these corrections numerically. Using the LECs as determined in [22][23] (and ignoring the quark mass renormalization of c_1, i.e. setting $c_1' = c_1$), we get

$$\delta m^{(4)} = (72.5 - 15.1 + 0.4 - 0.4) \text{ MeV}\,, \tag{B.12}$$

which shows that with the exception of the (undetermined) b_0–term (hidden in c_1'), the fourth order corrections are tiny. The general structure of the fourth order contribution to the nucleon mass was already given by Kallen [24], but we do not agree with some of her coefficients.

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